A Method for the Accurate Measurement of the Noise Temperature Ratio of Microwave Mixer Crystals*

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Summary—For the precise measurement of noise temperature ratio of a microwave mixer crystal it is common practice to employ a Roberts coupling network in order to make the Y factor independent of crystal conductance. It is shown here that a number of errors are introduced in this method, the chief of which is failure to consider the effect of transit time loading. It is also shown that the use of cathode lead inductance leads to a much improved measurement of noise temperature ratio.

Introduction

As IS WELL KNOWN, the noise temperature ratio (t) of a mixer crystal cannot be measured directly because of the minute amount of noise power which it generates. When placed across the input of an IF amplifier an auxiliary ratio, called the Y factor, can be measured with relative ease. It is related to t by

$$Y = \frac{t-1}{F_2'} + 1, \quad (1)$$

if $F_2'/F_{2s}' = G_{2s}/G_2$.

Since measurements of Y involve the noise figure of the amplifier, the measured value will be a function, not only of t, but of the conductance of the crystal as well.[1] It is common practice to place between the crystal and amplifier a coupling network, known as a Roberts line, which, to a fair approximation, makes the Y factor independent of crystal conductance. It has been shown[2] that, with this device,

$$Y = \frac{1}{F_{2s}'}\left[\frac{4\left(\frac{2pt}{1+p^2}+1\right)}{\left(\frac{2p}{1+p^2}+1\right)^2+\left(\frac{1-p^2}{1+p^2}\right)^2} - 2\right] + 1, \quad (2)$$

which reduces to

$$Y = \frac{t-1}{F_{2s}'\frac{(1+p)^2}{4p}} + 1. \quad (3)$$

It is shown in Appendix I that

$$\frac{F_2'}{F_{2s}'} = \frac{(1+p)^2}{4p} = \frac{G_{2s}}{G_2}, \quad (4)$$

so that

$$Y = \frac{t-1}{F_2'} + 1,$$

and the use of the Roberts line is justified.

Sources of Error

There are a number of sources of error in the above method for the determination of t.

(a) If Y is plotted against $\log p$ over the range $p=1/2$ to $p=2$, which includes practically all of the crystals of interest, a straight line, indicating constant Y is obtained only for $t=1$; the graph is symmetrically curved about the $p=1$ axis for all other values of t.[2] This means that a measured value of Y gives an uncertain value of t, unless the conductance of the crystal is measured independently.

(b) The form of equation (1) indicates that a set of Y versus $\log p$ curves for a range of values of t will be crowded closer together for higher values of F_2', making the attainable precision much less when reading t from such a chart.

(c) The theoretical development which was used in deriving the above fails to take into consideration the effect of transit-time loading conductance, or input grid noise, which is equivalent to assuming for it a noise temperature ratio of unity, whereas it has been shown to have a value of about 5.[3,4]

Transit-Time Loading Effect

The effect on the Y factor due to inclusion of the above factor may be developed from a consideration of Fig. 1. The conductance g_x is the sum of g_c, the IF conductance of the circuit losses at the input to the first tube, and g_t, the conductance due to transit-time loading. The mean-squared-noise current due to these conductances in an incremental frequency band df is given by the following equation

$$\overline{di_x^2} = 4KT_0df(g_c + \alpha g_t),$$

where α is the noise temperature ratio of g_t.

$$\therefore\ \overline{di_x^2} = 4KT_0df\left(\frac{g_c+\alpha g_t}{g_c+g_t}\right)(g_c+g_t)$$
$$= 4KT_0 s g_x df$$

* This work was performed under the Defence Research Board of Canada, Research Contract X-33.

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[1] H. C. Torrey and C. A. Whitmer, "Crystal Rectifiers"; McGraw-Hill, Radiation Laboratory Series, Vol. 15, pp. 25–31; 1948.

[2] *Ibid.*, p. 225.

[3] H. Wallman, A. B. Macnee, and C. P. Gadsden, "A low-noise amplifier," Proc. IRE, vol. 36, p. 700; June, 1948.

[4] G. E. Valley and H. Wallman, "Vacuum Tube Amplifiers," McGraw-Hill, Radiation Laboratory Series, Vol. 18, p. 638, 1948.

where s is the noise temperature ratio of g_x, and

$$s = \frac{g_c + \alpha g_t}{g_c + g_t}. \qquad (5)$$

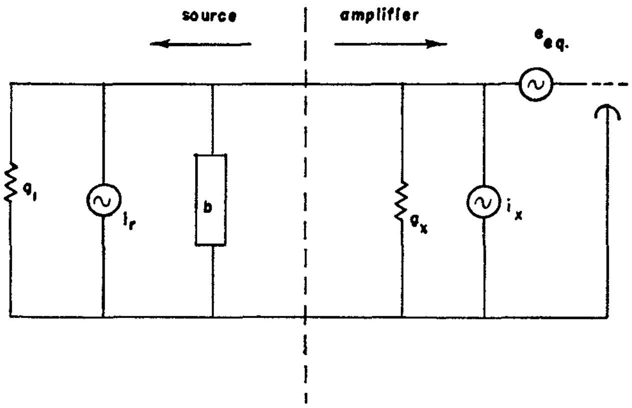

Fig. 1

The mean-squared-noise current at the input, due to i_1 and i_x is

$$\overline{di_T^2} = 4KT_0 df(tg_1 + sg_x).$$

This current gives rise to a voltage at the grid

$$\overline{de_T^2} = \frac{4KT_0 df(tg_1 + sg_x)}{(g_1 + g_x)^2 + b^2}. \qquad (6)$$

Mean-squared-noise voltage due to shot effect, referred to the grid, is

$$\overline{de_{eq}^2} = 4KT_0 df R_{eq}, \qquad (7)$$

and total noise voltage at the grid is

$$\overline{de_g^2} = \overline{de_T^2} + \overline{de_{eq}^2}.$$

If the power gain U of the amplifier is defined by

$$U = \frac{\text{power delivered to a given load}}{\text{power developed at the grid}},$$

then output power dN_0 in a band df is given by

$$dN_0 = U(g_x\overline{de_T^2} + g_x\overline{de_{eq}^2}), \qquad (8)$$

where the second term is a constant of the amplifier, as seen in (7). Substituting (6) and (7) in (8),

$$dN_0 = 4UKT_0\left[\frac{(tg_1 + sg_x)g_x}{(g_1 + g_x)^2 + b^2} + R_{eq}g_x\right]df$$

and

$$N_0 = 4KT_0\frac{(tg_1 + sg_x)g_x}{(g_1 + g_x)^2 + b^2}\int_0^\infty U df$$

$$+ 4KT_0 g_x\int_0^\infty UR_{eq}df, \qquad (9)$$

where it is assumed that the bandwidth of the following amplifier is narrow enough that the quantities taken outside the integral remain sensibly constant.

With a standard resistor on the input, $g_1 = g_a$, $b = 0$, $t = 1$, so that

$$N_{0s} = 4KT_0\frac{(g_a + sg_x)g_x}{(g_a + g_x)^2}\int_0^\infty U df + 4KT_0 g_x\int_0^\infty UR_{eq}df,$$

and

$$4KT_0 g_x\int_0^\infty UR_{eq}df$$

$$= N_{0s} - 4KT_0\frac{(g_a + sg_x)g_x}{(g_a + g_x)^2}\int_0^\infty U df. \qquad (10)$$

Now,

$$N_{0s} = F_{2s}'KT_0\int_0^\infty G_{2s}df.$$

To evaluate G_{2s}, note that available power from an IF signal generator of internal conductance g_1 and short-circuit current i_1 is

$$S_1 = \frac{i_1^2}{4g_1};$$

while the output power as read by a meter,

$$S_0 = e_g^2 g_x U = \frac{i_1^2 g_x}{(g_1 + g_x)^2 + b^2}U.$$

The available power gain of the network is

$$G_2 = \frac{S_0}{S_1} = \frac{4g_1 g_x}{(g_1 + g_x)^2 + b^2}U.$$

For a standard conductance, $g_1 = g_a$, $b = 0$, so that

$$G_{2s} = \frac{4g_a g_x}{(g_a + g_x)^2}U \qquad (11)$$

$$\therefore\ N_{0s} = F_{2s}'\cdot 4KT_0\frac{g_a g_x}{(g_a + g_x)^2}\int_0^\infty U df. \qquad (12)$$

Combining (9), (10), and (12),

$$Y = \frac{N_0}{N_{0s}} = \frac{1}{F_{2s}'}\left\{\frac{(tg_1 + sg_x)(g_a + g_x)^2}{g_a[(g_1 + g_x)^2 + b^2]} - \left(1 + \frac{sg_x}{g_a}\right)\right\} + 1.$$

Identifying g_x with g_a, and applying the conditions for a properly matched Roberts line,

$$Y = \frac{1}{F_{2s}'}\left[\frac{4\left(\frac{2pt}{1+p^2} + s\right)}{\left(\frac{2b}{1+p^2} + 1\right)^2 + \left(\frac{1-p^2}{1+p^2}\right)^2} - (1+s)\right] + 1. \qquad (13)$$

In this expression, when $t = s$

$$Y = \frac{s-1}{F_{2s}'} + 1, \text{ independent of } p, \qquad (14)$$

so that in a Y versus log p chart, the $t = s$ curve is a straight line, rather than $t = 1$. A typical set of curves is shown in Fig. 2, calculated for $F_{2s}' = 4.18$ db, which is the optimum noise figure with a 6AK5 as the input tube.

With an auxiliary method for measurement of p the relation given by (13) gives a satisfactory value of t, but practical considerations show that it is difficult to obtain the correct adjustment for the Roberts line. If the output noise could be made independent of p for $t=1$, the line could be set up simply by adjusting the components to the desired values with a bridge, followed by fine adjustment of the assembled circuit for constant power with varied source conductance.

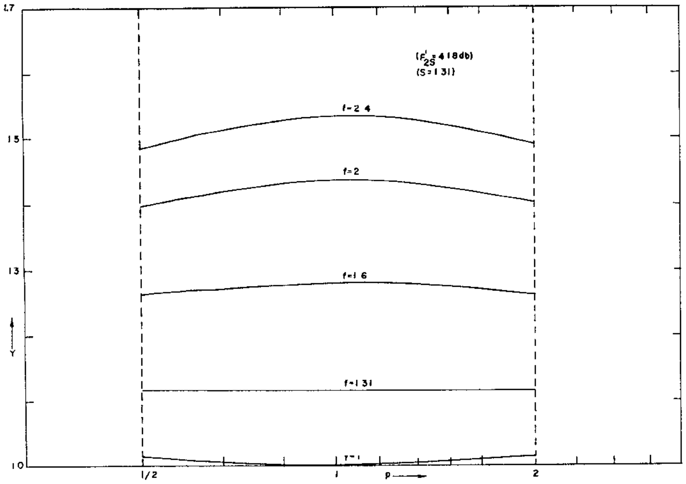

Fig. 2

Y Factor with the Roberts Line and Cathode Lead Inductance

If cathode lead inductance is introduced into the first stage of the preamplifier, a noiseless conductance[5] appears across its input terminals which may be varied at will by suitably changing the amplifier parameters. Thus the condition $s=1$ could be achieved by adjusting this noiseless conductance, until

$$g_f + g_t = \alpha g_t,$$

where

g_f = noiseless conductance due to feedback,

g_t = transit time loading conductance,

α = noise temperature ratio of g_t,

and the combined conductance g_f+g_t has an effective noise temperature ratio of unity.

However, when cathode lead inductance is introduced, the shot noise appearing in the plate circuit is no longer a constant of the tube, but is dependent on the input loading of the amplifier, and so varies with variation in source conductance. It may be shown that both shot noise and induced grid noise appearing in the output circuit are decreased by the factor

$$\frac{(g_1 + g_x)^2 + b^2}{(g_1 + g_x + g_f)^2 + b^2},$$

and, as a consequence, the equivalent circuit shown in Fig. 3 may be used. The mean-squared-noise voltage appearing at the grid is then

[5] *bid.*, pp. 673, 675.

$$\overline{de_g^2} = 4KT_0 df\left[\frac{tg_1 + sg_x}{(g_1 + g_x)^2 + b^2} + R_{eq}\right] \cdot \left[\frac{(g_1 + g_x)^2 + b^2}{(g_1 + g_x + g_f)^2 + b^2}\right].$$

The output meter reading gives

$$dN_0 = 4\,\underline{U}KT_0\left[\frac{(tg_1+sg_x)(g_f+g_x)}{(g_1+g_x)^2+b^2} + R_{eq}(g_f+g_x)\right]df, \quad (15)$$

where

$$\underline{U} = U\frac{(g_1 + g_x)^2 + b^2}{(g_1 + g_x + g_f)^2 + b^2}.$$

Note that $\underline{U}$ is *not* a constant of the amplifier, but is dependent on the source admittance.

Integrating,

$$N_0 = 4KT_0\frac{(tg_1 + sg_x)(g_f + g_x)}{(g_1 + g_x)^2 + b^2}\int_0^\infty \underline{U}df + 4KT_0(g_f + g_x)\int_0^\infty \underline{U}R_{eq}df, \quad (16)$$

assuming again that quantities outside integral remain sensibly constant over passband of amplifier.

For the standard conductance

$$N_{0s} = 4KT_0\frac{(g_a + sg_x)(g_f + g_x)}{(g_a + g_x)^2}\int_0^\infty \underline{U}'df + 4KT_0(g_f + g_x)\int_0^\infty \underline{U}'R_{eq}df,$$

where

$$\underline{U}' = \frac{(g_a + g_x)^2}{(g_a + g_x + g_f)^2}U$$

$$\therefore\ 4KT_0(g_f + g_x)\int_0^\infty \underline{U}'R_{eq}df = N_{0s} - 4KT_0\frac{(g_a + sg_x)(g_f + g_x)}{(g_a + g_x)^2}\int_0^\infty \underline{U}'df. \quad (17)$$

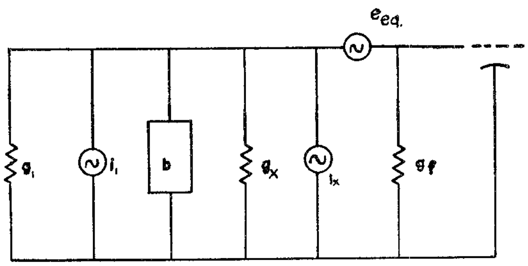

Fig. 3

With an IF signal generator of internal conductance g_a and short-circuit current i_1, output meter reading is

$$S_0 = \frac{i_1^2(g_f + g_x)}{(g_a + g_x + g_f)^2}U = \frac{i_1^2(g_f + g_x)}{(g_a + g_x)^2}\underline{U}'$$

and

$$S_1 = \frac{i_1^2}{4g_a}$$

$$\therefore\ G_{2s} = \underline{U'}\frac{4g_a(g_f + g_x)}{(g_a + g_x)^2}$$

$$\therefore\ N_{0s} = F_{2s}'KT_0\int_0^\infty G_{2s}df$$

$$= F_{2s}'\cdot 4KT_0\frac{g_a(g_f + g_x)}{(g_a + g_x)^2}\int_0^\infty \underline{U'}df. \quad (18)$$

Recalling the defining relations for $\underline{U}$ and $\underline{U'}$

$$\underline{U} = \gamma\,\underline{U'} \quad (19)$$

where

$$\gamma = \frac{(g_1 + g_x)^2 + b^2}{(g_1 + g_x + g_f)^2 + b^2}\cdot\frac{(g_a + g_x + g_f)^2}{(g_a + g_x)^2}. \quad (20)$$

Using (16) to (20):

$$Y = \frac{N_0}{N_{0s}} = \frac{1}{F_{2s}'}\left[\frac{(tg_1 + sg_x)(g_a + g_x + g_f)^2}{g_a(g_1 + g_x + g_f)^2 + b^2} - \gamma\left(1 + \frac{sg_x}{g_a}\right)\right] + \gamma. \quad (21)$$

Setting $g_x + g_f = g_a$, and using the matching conditions for the Roberts line,

$$Y = \frac{1}{F_{2s}'}\left[\frac{4\left(\frac{2pt}{1 + p^2} + \frac{sg_x}{g_a}\right)}{\left(\frac{2p}{1 + p^2} + 1\right)^2 + \left(\frac{1 - p^2}{1 + p^2}\right)^2} - \gamma\left(1 + \frac{sg_x}{g_a}\right)\right] + \gamma, \quad (22)$$

where

$$\gamma = \frac{4}{\left(1 + \frac{g_x}{g_a}\right)^2}\left[\frac{\left(\frac{2p}{1 + p^2} + \frac{g_x}{g_a}\right)^2 + \left(\frac{1 - p^2}{1 + p^2}\right)^2}{\left(\frac{2p}{1 + p^2} + 1\right)^2 + \left(\frac{1 - p^2}{1 + p^2}\right)^2}\right]. \quad (23)$$

In order that (1), (2), and (22) may be identical, the necessary conditions are

(a) $\gamma = 1$

(b) $\frac{sg_x}{g_a} = 1$

(c) $g_a = g_x + g_f$ or, $g_f = (\alpha - 1)g_t$.

From (c) it is noted that the desired feedback conductance is a constant for any given transit time loading g_t, for which the noise temperature ratio is α.

Since shot noise and induced grid noise are decreased by the same factor when cathode lead inductance is used in the amplifier, the noise figure of the amplifier must remain as given in (36), with s instead of unity as the noise temperature ratio of the transit-time loading, *i.e.*,

$$F_{2s}' = 1 + \frac{sg_x}{g_a} + \frac{R_{eq}}{g_a}(g_a + g_x)^2,$$

which, with condition (b), becomes

$$F_{2s}' = 2 + \frac{R_{eq}}{g_a}(g_a + g_x)^2 \quad (24)$$

It may easily be shown that the source conductance for optimum noise figure is $g_a = 1/2\ g_f$ and the optimum value of F_{2s}' is 2. The imposed conditions make the attainment of this optimum impossible, but (24) shows that F_{2s}' decreases as g_a is decreased. Table I shows calculated values for two typical cases, using a strapped 6AK5, with the parallel figures, without feedback, shown for comparison.

TABLE I

ΔN_{01} from $p=1$ to $p=0$ for $t=1$	Feedback		No Feedback	
	g_a	F_{2s}'	g_a	F_{2s}'
0.8 per cent	416 μmhos	3.15 db	5,000 μmhos	6.77 db
0.1 per cent	90	4.10	1,700	10.00

It will be seen that for a given accuracy in setting up the line, the amplifier with cathode lead inductance has the distinct advantage that its noise figure is from 3.5 to 6 db lower than that of a similar amplifier without feedback.

The condition of $\Delta N_{01} = 0.1$ per cent is one that can be attained with a practical amplifier, i.e., the Y curve which it generates for $t=1$ can be made to lie within 0.1 per cent of the curve given by equation (1) at the extreme $p=0$, and will coincide with it at $p=1$ ($\gamma_{p=1}=1$). In addition, as t increases, the per cent variation between the two sets of curves decreases.

The discrepancies between (13) and (22), and (1) may be explained in terms of the variation of amplifier noise figure as a function of source conductance for these two amplifier configurations.

From the simple noise theory,

$$Y_{t=1} = \frac{F_2'}{F_{2s}'}\cdot\frac{G_2}{G_{2s}},$$ assuming that bandwidth remains constant.

But

$$\frac{G_2}{G_{2s}} = \frac{4p}{1 + p^2},$$ as shown in Appendix I

$$\therefore\ Y_{t=1} = \frac{4p}{1 + p^2}\frac{F_2'}{F_{2s}'}. \quad (25)$$

From (13)

$$Y_{t=1} = \frac{s - 1}{F_{2s}'}\frac{(1 - p)^2}{(1 + p)^2} + 1. \quad (26)$$

Combining (25) and (26):

$$\frac{F_2'}{F_{2s}'} = \frac{(1 + p)^2}{4p}\left[\frac{(s - 1)}{F_{2s}'}\frac{(1 - p)^2}{(1 + p)^2} + 1\right]. \quad (27)$$

Thus, for a normal amplifier with a Roberts line input network, (1) is satisfied only when $p=1$, or $s=1$ (which is physically impossible).

From (22), when $sg_x=g_a$,

$$Y_{t=1} = \frac{2(1-\gamma)}{F_{2s}'} + \gamma. \tag{28}$$

Combining (25) and (28):

$$\frac{F_2'}{F_{2s}'} = \frac{1+p^2}{4p}\left[\frac{2(1-\gamma)}{F_{2s}'} + \gamma\right]. \tag{29}$$

Thus, for an amplifier with cathode lead inductance and a Roberts line network, (1) is satisfied only for $p=1$, or $F_{2s}'=2$, (which is also physically impossible).

Although the conditions expressed in (27) and (29) are both unattainable, the square bracket term in (29) will remain much closer to unity than will the corresponding term in (27), as p undergoes variation. Thus curves generated by (22) will approximate those of (1) more closely than will those generated by (13).

The Determination of p

Usually a diode noise source is included in a crystal noise measuring set to provide a source of known IF noise power for the measurement of the IF amplifier noise figure. When a Roberts line is used in the amplifier circuit, one measurement with the diode noise source, in addition to the Y-factor measurement, will suffice to determine the IF conductance of the crystal.

The output noise power of the crystal-amplifier combination, when the crystal is supplied with power from a microwave local oscillator and the diode noise source is switched off, is given by

$$N_0 = KT_0G_2B_{1+2}(F_2' + t - 1).$$

The output noise power from the crystal-amplifier combination, when the crystal is supplied with the *same amount* of microwave power, and the diode noise source is switched on, is given by[6]

$$N_{0N} = KT_0G_2B_{1+2}\left(F_2' + t - 1 + \frac{20I}{g_1'}\right),$$

where I is the dc plate current of the diode.

Note that only the IF component of the diode plate current may flow through the crystal; the dc current must be blocked, otherwise the IF conductance of the crystal will be altered.

If the dc current through the diode is increased until

$$N_{0N} = \beta N_0$$

then

$$\frac{20I}{g_1'} = (\beta - 1)(F_2' + t - 1),$$

and

$$\frac{1}{g_1'} = (\beta - 1)\left(\frac{F_2'}{20I} + \frac{t-1}{F_2'}\cdot\frac{F_2'}{20I}\right);$$

[6] "Crystal Rectifiers," p. 226.

but, from (1)

$$\frac{t-1}{F_2'} = Y - 1,$$

$$\therefore\ \frac{1}{g_1'} = (\beta - 1)\frac{YF_2'}{20I}.$$

Again, using the relations

$$g_1' = pg_s \quad\text{and}\quad F_2' = F_{2s}'\frac{(1+p)^2}{4p}$$

$$\frac{1}{pg_s} = (\beta - 1)\frac{YF_{2s}'(1+p)^2}{80pI},$$

$$\therefore\ p = \sqrt{\frac{80}{g_sF_{2s}'(\beta - 1)}}\sqrt{\frac{I}{Y}} - 1. \tag{30}$$

Now, if β is made equal to 2, that is, if the noise output meter reading is doubled,

$$p = C_1\sqrt{\frac{I}{Y}} - 1, \tag{31}$$

where

$$C_1 = \sqrt{\frac{80}{g_sF_{2s}'}},$$

and is a constant of the amplifier.

Procedure for Measurement of t

(1) The crystal to be tested is placed in the crystal holder and supplied with some given amount of local oscillator power.

(2) With the crystal connected to the input of the amplifier, the gain is turned up to obtain full-scale deflection on the output meter.

(3) The amplifier input is then switched to a standard resistor and the output meter deflection is noted.

(4) The amplifier input is then switched back to the crystal and the gain turned down to obtain one-half of full-scale deflection on the meter.

(5) The diode noise source is turned on, and the dc plate current I, necessary to increase the output noise to full-scale deflection, is noted.

Steps (2) and (3) yield the Y factor directly, which, combined with I from step (5) yields the required value of t.

Attainable Accuracy in the Measurement of t

It is shown in Appendix II that the error arising in the measurement of t due to the assumptions made in the theory is less than 0.2 per cent for an amplifier bandwidth of 0.25 mc, and would be less for a smaller bandwidth. Also, with cathode lead inductance, the Y-factor curves will lie within 0.1 per cent of those generated by (1), even at the extremes of $p=0$ and $p=\infty$; also the noise figure of the amplifier will remain near the theoretical minimum. In measurement of p the only basic source of error is in use of (1) rather than (22), so that the error in p is of the order of 0.1 per cent.

From the above, it is apparent that any uncertainty in the measured value of t is much more likely to arise from experimental errors than from errors inherent in the theory. Actual measurements are confined to the reading of an output power meter and a meter indicating diode current. Assume that these are spotlight galvanometers with a full-scale deflection of 100 divisions, and that these meters may be *set* to a predetermined deflection to within 0.1 division; and may be *read* for a deflection imposed by the conditions of experiment to within 0.2 division. Since either instrument may be used at nearly full scale by means of precision resistor shunts, any given deflection may be set up to ±0.1 per cent and any deflection may be read to ±0.2 per cent.

The Y factor is the ratio of two meter readings, one *set* and the other *read*, so that

$$\left(\frac{\Delta Y}{Y}\right)_{max} = 0.3 \text{ per cent.} \tag{32}$$

The percentage variation in p may be found from

$$p = \sqrt{\frac{80I}{F_{2s}' g_s Y}} - 1.$$

With a standard conductance g_s on the input, $Y=1$, $p=1$, and

$$F_{2s}' = \frac{20I_1}{g_s},$$

$$\therefore\ p = 2\sqrt{\frac{I}{I_1 Y}} - 1.$$

Now

$$\Delta p = \left(\frac{\partial p}{\partial Y}\right)\Delta Y + \left(\frac{\partial p}{\partial I}\right)\Delta I + \left(\frac{\partial p}{\partial I_1}\right)\Delta I_1$$

$$\therefore\ \frac{\Delta p}{p+1} = \frac{1}{2}\left(\frac{\Delta Y}{Y} + \frac{\Delta I}{I} + \frac{\Delta I_1}{I_1}\right).$$

From above,

$$\left(\frac{\Delta Y}{Y}\right)_{max} = \pm\ 0.3 \text{ per cent,}$$

$$\left(\frac{\Delta I}{I}\right) = \left(\frac{\Delta I_1}{I_1}\right) = \pm\ 0.2 \text{ per cent}$$

$$\therefore\ \left(\frac{\Delta p}{p+1}\right)_{max} = \pm\ 0.35 \text{ per cent}$$

and

$$\left(\frac{\Delta p}{p}\right)_{max} = \pm\left(\frac{p+1}{p}\right) 0.35 \text{ per cent.} \tag{33}$$

Values of $(\Delta p/p)_{max}$ are shown in Table II for a range of values of p.

From the above it is seen that for any given t the error due to measurement of p will be of importance only for crystals of extremely small conductance.

TABLE II

p	$(\Delta p/p)_{max}$
1/10	3.85 per cent
1/2	1.05
1	0.70
2	0.53
10	0.39

The value of t is given by

$$t = (Y-1)F_{2s}' \frac{(1+p)^2}{4p} + 1,$$

and, for purposes of measurement, Y and p are independent variables.

$$\therefore\ \Delta t = \left(\frac{\partial t}{\partial p}\right)_Y \Delta p + \left(\frac{\partial t}{\partial Y}\right)_p \Delta Y$$

$$\left(\frac{\partial t}{\partial p}\right)_Y = (Y-1)F_{2s}'\left(\frac{p^2-1}{4p^2}\right) = \frac{t-1}{p}\cdot\frac{p^2-1}{(1+p)^2}$$

$$\left(\frac{\partial t}{\partial Y}\right)_p = F_{2s}' \frac{(1+p)^2}{4p}$$

$$\therefore\ \Delta t = (t-1)\frac{p^2-1}{(1+p)^2}\frac{\Delta p}{p} + F_{2s}'\frac{(1+p)^2}{4p}\Delta Y,$$

and

$$\frac{\Delta t}{t} = \left(\frac{t-1}{t}\right)\frac{p^2-1}{(1+p)^2}\frac{\Delta p}{p} + F_{2s}'\frac{(1+p)^2}{4p}\left(\frac{Y}{t}\right)\frac{\Delta Y}{Y}. \tag{34}$$

From this last equation:

(a) Error in t due to error in measurement of p

$$= 0, \quad \text{when } t = 1,$$

$$= \frac{p^2-1}{(1+p)^2}\left(\frac{\Delta p}{p}\right), \quad \text{when } t \gg 1,$$

and is, of course, zero for any t, when $p=1$.

(b) Error in t due to error in measurement of Y

$$= F_{2s}'\frac{(1+p)^2}{4p}\left(\frac{\Delta Y}{Y}\right), \quad \text{when } t = 1,$$

$$= \frac{\Delta Y}{Y}, \text{ for all values of } p, \text{ when } t \gg 1.$$

Using (34) together with (32) and (33), the value of $\Delta t/t$ for any p and t may be calculated. Table III shows sets of values for typical and extreme values of p and t.

As a general result it may be stated that the maximum experimental error in the range of conductances, $p=1/2$ to $p=2$, is less than 1 per cent, and the probable error for this range is ±0.6 per cent.

TABLE III

MAXIMUM MEASUREMENT ERROR IN t EXPRESSED AS A PERCENTAGE OF t FOR A RANGE OF VALUES OF p AND OF t

t	Error contributed by p term	Error contributed by Y term	$(\Delta t/t)_{max}$
	(a) $p=1/10$		
	per cent	per cent	per cent
1	0	2.32	2.32
2	1.58	1.31	2.89
3	2.10	0.97	3.07
$\gg 1$	3.15	0.30	3.45
	(b) $p=10$		
1	0	2.32	2.32
2	0.16	1.31	1.47
3	0.21	0.97	1.18
$\gg 1$	0.32	0.30	0.62
	(c) $p=1/2$		
1	0	0.87	0.87
2	0.18	0.58	0.76
3	0.23	0.49	0.72
$\gg 1$	0.35	0.30	0.65

APPENDIX I

Amplifier Noise Figure and Gain with Assumed Noise Temperature Ratio of Unity for Transit-Time Loading

(a) Referring to Fig. 1, the mean-squared-noise current at the input, due to i_1 and i_x is

$$\overline{di_T^2} = 4KT_0df(g_1 + g_x),$$

and the mean-square noise current due to e_{eq} is

$$\overline{di_{eq}^2} = 4KT_0dfR_{eq}(g_1 + g_x)^2 + b^2,$$

$$\therefore \overline{di_q^2} = \overline{di_T^2} + \overline{di_{eq}^2}$$
$$= 4KT_0df(g_1 + g_x) + R_{eq}[(g_1 + g_x)^2 + b^2].$$

The source produces noise current

$$\overline{di_1^2} = 4KT_0df \cdot g_1$$

$$\therefore F_2 = 1 + \frac{g_x}{g_1} + \frac{R_{eq}}{g_1}[(g_1 + g_x)^2 + b^2]. \quad (35)$$

With a standard resistor on the input of the Roberts line, $g_1 = g_a$, $b=0$

$$\therefore F_{2s} = 1 + \frac{g_x}{g_a} + \frac{R_{eq}}{g_a}(g_a + g_x)^2. \quad (36)$$

Applying the matching conditions for the Roberts line,

(b) Available power from an IF signal generator of internal conductance g_1 and short-circuit current i_1 is

$$S_1 = \frac{i_1^2}{4g_1}.$$

Output meter reading

$$S_0 = e_g^2 g_x U$$
$$= \frac{i_1^2 g_x}{(g_1 + g_x)^2 + b^2} U.$$

Available power gain of the network is

$$G_2 = \frac{S_0}{S_1} = \frac{4g_1g_x}{(g_1 + g_x)^2 + b^2} U.$$

Available power gain with a standard conductance g_s on the input, where $g_1 = g_a$, $b=0$,

$$G_{2s} = \frac{4g_ag_x}{(g_a + g_x)^2} U$$

$$\therefore \frac{G_2}{G_{2s}} = \frac{g_1}{g_a} \frac{(g_a + g_x)^2}{(g_1 + g_x)^2 + b^2}.$$

Applying the Roberts line matching conditions,

$$\frac{G_2}{G_{2s}} = 4 \frac{\dfrac{2p}{1 + p^2}}{\left(\dfrac{2p}{1 + p^2} + 1\right)^2 + \left(\dfrac{1 - p^2}{1 + p^2}\right)^2}$$

$$\therefore \frac{G_2}{G_{2s}} = \frac{4p}{(1 + p)^2}. \quad (38)$$

APPENDIX II

Practical Aspects of the Roberts line

Inherent in the theoretical treatment which has been presented here are two main assumptions:

(1) that the bandwidth of the crystal-amplifier combination is determined by the bandwidth of the amplifier alone, and is narrow enough that $F_2 = F_2'$;

(2) that the output conductance and susceptance of the Roberts line may be considered constant over the pass band of the amplifier, so that

$$g_1 = g_a \frac{2p}{1 + p^2}, \quad b = \pm g_a \frac{1 - p^2}{1 + p^2}, \quad g_x = g_a$$

$$\frac{F_2}{F_{2s}} = \frac{\left(1 + \dfrac{1 + p^2}{2p}\right) + R_{eq}g_a \dfrac{1 + p^2}{2p}\left[\left(\dfrac{2p}{1 + p^2} + 1\right)^2 + \left(\dfrac{1 - p^2}{1 + p^2}\right)^2\right]}{2 + 4R_{eq}g_a}$$

$$\therefore \frac{F_2}{F_{2s}} = \frac{(1 + p)^2}{4p}. \quad (37)$$

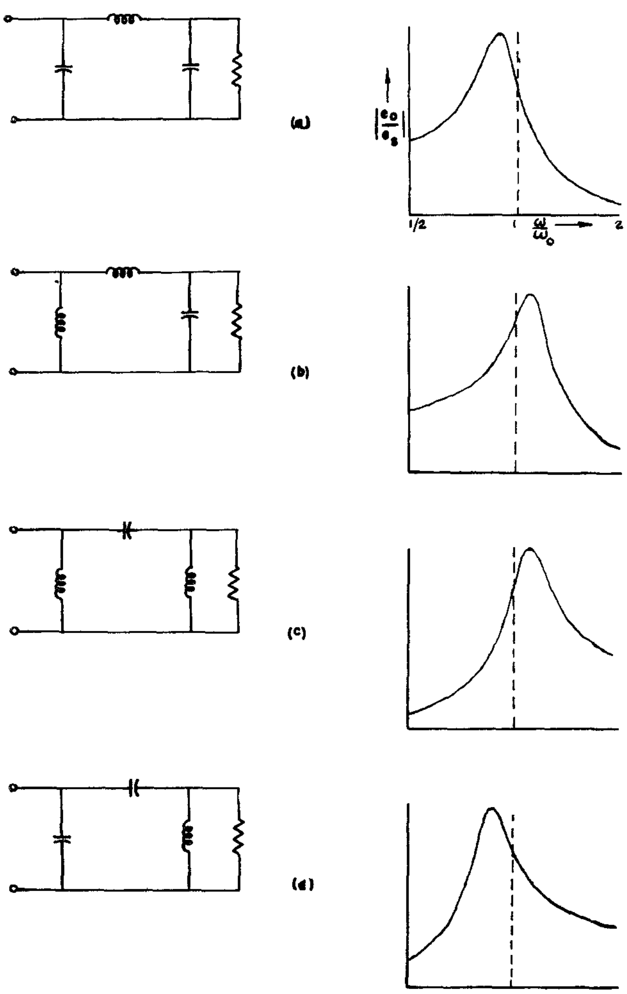

Fig. 4

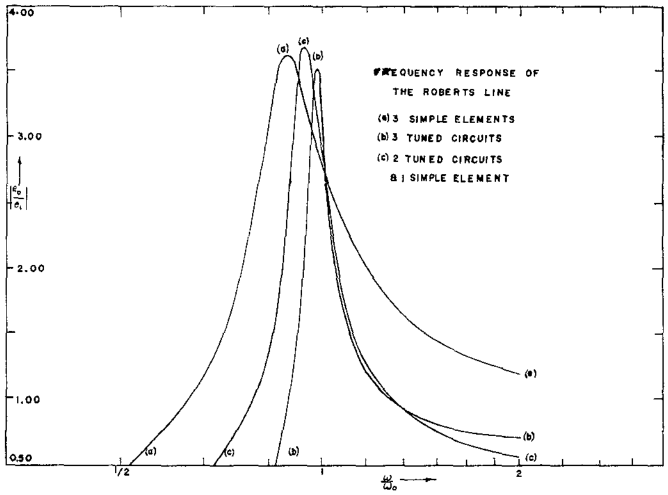

Fig. 5

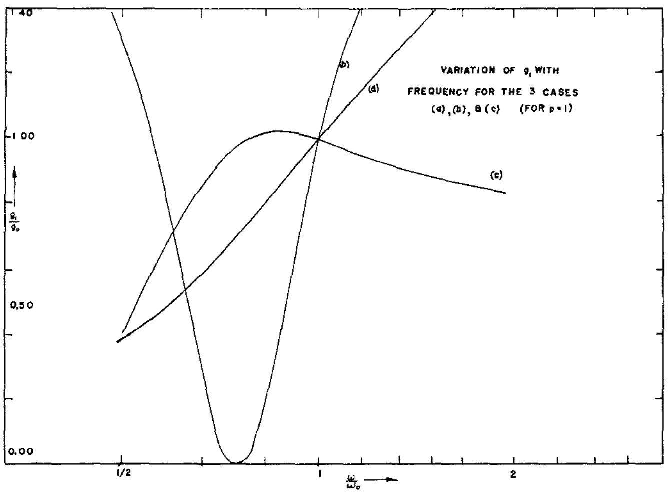

Fig. 6

$$\frac{\eta \int U df}{\int \eta U df} = 1,$$

where

$$\eta = \frac{4(tg_1 + g_a)g_a}{(g_1 + g_a)^2 + b^2}.$$

Since the Roberts line is a network of three reactance components, it might be expected to have a critical bearing on bandwidth of the circuit. There are four possible Roberts line configurations for any desired impedance step-up ratio, shown in Fig. 4, together with sketches of their frequency response. There may be two coils and one condenser, as in (b), (c), or two condensers and one coil, as in (a), (d). To minimize coupling among the components, the latter two are preferable; and to minimize coupling between the network and the amplifier it is preferable to have the single coil in the output arm, as in (d), rather than in the series arm, as in (a). In addition, the response curves show that in (a) the operating frequency is on the side of greater slope, while in (d) it is on the side of less slope, so that again the configuration in (d) is the preferred one.

In practice it is impossible to have simple elements as the three components of the line. The crystal mixer is attached to a plug across the input terminals of the line, and the capacitance of mixer and plug is, by itself, too great for the desired input component, so that it must be shunted by an inductance to reduce its value at 30 mc to the desired value. Also, the output (inductance) arm must include the input capacitance of the amplifier. This means that the input and output arms of the line must be tuned circuits, while the series arm may or may not be a tuned circuit. In Fig. 5 are shown the voltage transfer characteristics for the three configurations: (a) all three components simple elements; (b) all three components tuned circuits; (c) a simple component in the series arm and a tuned circuit in each of the others.

The variation in output conductance as a function of frequency is shown for each of these three cases in Fig. 6. From Figs. 5 and 6 it is evident that case (c) is to be preferred, both with respect to frequency response and to variation in output conductance with frequency.

From Fig. 5 it is obvious that the Roberts line is not an extremely wide-band network, and it is necessary that amplifier bandwidth is sufficiently narrow to satisfy the two conditions stated at the beginning.

(a)
$$F_2' = \frac{\int F_2 G_2 df}{\int G_2 df},$$

which would reduce to $F_2' = F_2$, if F_2 were constant at all frequencies. Actually, it is not, but the ratio F_2'/F_2 approaches unity as the bandwidth is decreased.

Arbitrarily, a bandwidth of 0.25 mc was chosen at an IF frequency of 30 mc, which is less than 1 per cent bandwidth. Under these conditions calculated values of the ratio are:

$$\frac{F_{2s}'}{F_{2s}} = 1.00093, \text{ for } p = 1$$

$$\frac{F_2'}{F_2} = 1.00081, \text{ for } p = 2.$$

The error in assuming $F_2'/F_{2s}' = F_2/F_{2s}$ is much less than either of the above errors, since the error in F_2' and that in F_{2s}' are in the same direction.

(b) For same bandwidth of 0.25 mc calculations show

$$\frac{\eta \int U df}{\int \eta U df} = 1.0040, \text{ for } p = 1 \text{ and}$$

$$= 1.0025, \text{ for } p = 2.$$

In actual calculations of total noise power, $KT_0\eta\int Udf$ is added to a constant term arising from the shot effect; consequently the error in the total output noise power will be less than indicated by the values of the ratio shown above. Further, since the Y factor is the ratio of two such expressions, with their respective errors lying in the same direction, the error in the Y factor itself will be much less. From these considerations it would appear that the error in Y factor resulting from the second assumption is not greater than about 0.1 per cent.

Appendix III

List of Symbols Not Explicitly Defined in the Text

G_1 = conversion gain of the crystal.

G_2 = available-power gain of the IF amplifier and output meter when driven by a signal generator whose internal conductance is equal to that of the crystal.

G_{2s} = available-power gain of the IF amplifier and output meter when driven by a generator of internal conductance g_s.

F_2' = effective noise figure of the IF amplifier with a resistor on the input.

F_{2s}' = effective noise figure of the IF amplifier with a standard resistor on the input.

(*Note*—unprimed F symbols refer to the noise figure for an incremental frequency band df.)

N_0 = output noise power of the over-all network with a crystal on the input.

N_{0s} = output noise power of the over-all network with a standard resistor on the input.

N_{0N} = output noise power of the over-all network with a crystal on the input driven by a diode noise source.

g_1' = IF conductance of a crystal.

g_s = standard IF conductance.

g_1 = conductance at output of the Roberts line with a conductance g_1' on the input.

g_a = conductance at the output of the Roberts line with a standard conductance g_s on the input.

b = susceptance at the output of the Roberts line with a conductance g_1' on the input ($b = 0$ for $g_1' = g_s$).

Circuit Components in Dielectric Image Lines

D. D. KING†

Summary—Symmetry of dipole mode in a dielectric rod permits use of an image system. By replacing lower half of dielectric and its surrounding field with an image surface, support problem is eliminated. Resulting image provides structural convenience and also has very low loss, provided wave is allowed to occupy a cross section many wavelengths square. In millimeter region this is readily achieved. Possibilities of new types of circuit elements in this image system are explored. Combination of optical and waveguide techniques is a characteristic of resulting components. Properties of several transducers between image line and either rectangular waveguide or coaxial line are described. Attenuators, standing-wave detector, and various directional coupler types for image lines are also discussed.

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Introduction

The HE_{11} or dipole mode in a thin dielectric rod offers very low attenuation and freedom from higher modes. This mode possesses a plane of symmetry, and hence may be operated as an image system.[1] The cross section of an image line is shown in Fig. 1. Only the E field in the dielectric is indicated. The complete mode pattern is complicated, since all three components of E and H are present. The theory of the dipole mode

[1] D. D. King. "Dielectric Image Line" (letter), *Jour. Appl. Phys.*, vol. 23, p. 699; June, 1952.